FLOW DYNAMICS AND HEAT TRANSFER IN A ROTATING SLOT CHANNEL

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An analysis is presented of the change in the flow caused by the rotation, as well as of the effects of the flow change on the heat transfer.

There is a need to improve the cooling of rotors in electrical machines and of gas-turbine blades, and an important part in research in this area is played by the determination of flow characteristics and heat-transfer parameters in a rotating channel whose axis is perpendicular to the axis of rotation. The need for such research is due to the marked effects of the rotation on the hydraulic resistance of the channel and on the heat-transfer rate. Here we consider the flow of a viscous liquid in a rotating channel of simple shape formed by two parallel planes; this defines the major effects arising from the rotation and gives a quantitative evaluation of the effects of the rotation on the resistance and heat-transfer rate.

Consider a prismatic slot channel of constant height 2h that rotates uniformly with an angular velocity ω about an axis perpendicular to the planes forming the slot.

We introduce a Cartesian coordinate system Oxyz rigidly coupled to the channel and oriented in such a way that the Oy axis lies along the axis of rotation, while the Oz axis is parallel to the side walls of the channel and is directed along the flow, and the origin lies in the median plane of the channel.

We first consider the dynamic problem. We assume that the flow has stabilized (the relative-velocity vector is not dependent on the z coordinate), and we restrict consideration to the flow in the central part of the channel far from the side walls. We assume that the velocity vector is parallel to the planes forming the slot at each point in this region and therefore is a function of the y coordinate only.

Thus, the Navier–Stokes equations for the rotating Oxyz coordinate system take the form

$$\left.\begin{aligned} \nu \frac{d^2u}{dy^2} &= \frac{\partial \Pi}{\partial x} + 2\omega w, \\ 0 &= \frac{\partial \Pi}{\partial y}, \\ \nu \frac{d^2w}{dy^2} &= \frac{\partial \Pi}{\partial z} - 2\omega u. \end{aligned}\right\} \tag{1}$$

Here u and w are the projections of the velocity vector on the x and z axes, respectively, while $\Pi=\Pi(x, z)$ is the modified pressure, which is defined by

$$\Pi = \frac{p}{\rho} - \frac{\omega^2}{2}(x^2 + z^2).$$

It follows from (1) that Π should be a linear function of x and z, i.e.,

$$\Pi(x, z) = \Pi_0 + \alpha x + \beta z.$$

The boundary conditions at the upper and lower walls are put in the usual form:

$$u = w = 0 \quad \text{at} \quad y = \pm h. \tag{2}$$

The presence of the side walls is incorporated by setting the liquid flow as zero for any section of the channel parallel to the Oyz plane:

$$\int_{-h}^{h} u(y)\, dy = 0. \tag{3}$$

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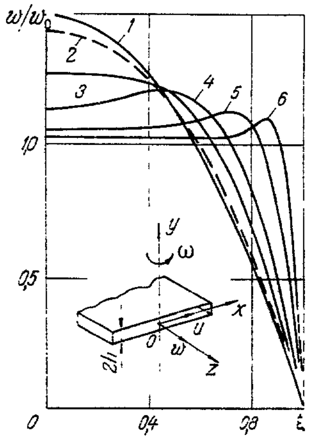

Fig. 1

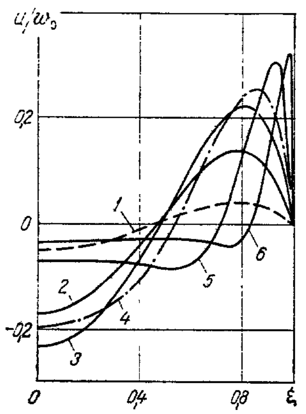

Fig. 2

Fig. 1. Curves for the longitudinal component of the velocity: 1) $\gamma=0$ ($\gamma=1$); 2) 2; 3) 3; 4) 4; 5) 8; 6) 16.

Fig. 2. Curves for the transverse component of the velocity: 1) $\gamma=1$; 2) 2; 3) 3; 4) 4; 5) 8; 6) 16.

The result is normalized by introducing the flow-rate velocity w_0, which is defined as the ratio of the flow rate Q along the z axis per unit width of channel to the height of the latter:

$$w_0 = \frac{Q}{2h} = \frac{1}{2h}\int_{-h}^{h} w(y)\,dy. \tag{4}$$

The solution to (1) that satisfies (2)-(4) can be put as

$$w/w_0 = C\{A[\operatorname{sh}(\gamma\xi)\sin(\gamma\xi) - \operatorname{sh}\gamma\sin\gamma] + B[\operatorname{ch}(\gamma\xi)\cos(\gamma\xi) - \operatorname{ch}\gamma\cos\gamma]\}, \tag{5}$$

$$u/w_0 = C[\sin 2\gamma - \operatorname{sh} 2\gamma - A\operatorname{ch}(\gamma\xi)\cos(\gamma\xi) + B\operatorname{sh}(\gamma\xi)\sin(\gamma\xi)], \tag{6}$$

where

$$\xi = y/h;\quad A = 2(\operatorname{ch}\gamma\sin\gamma - \operatorname{sh}\gamma\cos\gamma - 2\gamma\operatorname{sh}\gamma\sin\gamma);$$

$$B = 2(\operatorname{sh}\gamma\cos\gamma + \operatorname{ch}\gamma\sin\gamma - 2\gamma\operatorname{ch}\gamma\cos\gamma);$$

$$\gamma = \sqrt{\mathrm{Re}_\omega};\quad \mathrm{Re}_\omega = \frac{\omega h^2}{\nu};\quad C = 4\gamma/(A^2+B^2). \tag{7}$$

Figure 1 shows the distribution for the longitudinal component of the velocity w/w_0, as calculated from (5) for several values of γ; it is clear that the Coriolis force has only a slight influence on the distribution of this component if γ is small, and the distribution for $\gamma < 1$ is essentially that for laminar flow in a planar immobile channel. This follows also directly from (5) if written for small values of γ:

$$\frac{w}{w_0} = \frac{3}{2}\left[1 - \xi^2 + \frac{\gamma^4}{3150}(35\xi^6 - 105\xi^4 + 81\xi^2 - 11)\right] + O(\gamma^8).$$

In the range $1 < \gamma < 4$ there is a change in the structure of the profile, particularly flattening at the core; in the range $\gamma > 8$ the value of w/w_0 is virtually constant across the channel, apart from a small area near the walls. The speed at the core of the flow in that case is given approximately by

$$\frac{w}{w_0} \simeq 1 + \frac{1}{2\gamma}.$$

Figure 2 shows distributions for the transverse component of the velocity derived from (6) of several values of γ.

If γ is small, (6) can be put as

$$\frac{u}{w_0} = \frac{\gamma^2}{20}(6\xi^2 - 5\xi^4 - 1) + O(\gamma^6). \tag{8}$$

It follows from (8) that $u(0)/w_0 = -\gamma^2/20$; an interesting point is that the analogous value for the transverse velocity component at the center of a circular channel is $-\gamma^2/24$ [1]. The absolute value of $u(0)/w_0$ at first

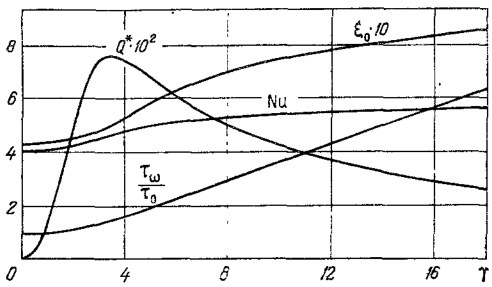

Fig. 3. Friction, transverse circulation flow, coordinates of the center of the transverse vortex, and Nusselt number as functions of the rotation parameter.

increases with γ, but a turning point occurs at $\gamma \simeq 3.4$. There is a substantial change in the distribution of u/w_0 in the range $3.4 < \gamma < 8$, particularly flattening at the core; the value of u/w_0 is virtually constant for $\gamma > 8$ throughout much of the cross section, the approximate value being $-0.5/(\gamma - 1)$; an exception is constituted by the region near the walls, where the velocity distribution is typical of an Eckman layer at a solid boundary [2]:

$$u/w_0 = \exp[-\gamma(1-\xi)] \sin[\gamma(1-\xi)], \tag{9}$$

$$w/w_0 = 1 - \exp[-\gamma(1-\xi)] \cos[\gamma(1-\xi)], \tag{10}$$

where (9) and (10) are derived from (5) and (6) as $\gamma \to \infty$.

Figure 3 shows the γ dependence of the coordinate ξ_0 of the point at which the component u (in the upper half of the channel) changes sign; (8) readily shows that for $\gamma \ll 1$ we have $\xi_0 = \sqrt{0.2}$, which coincides with the center of the eddy representing the secondary flow for channels of circular [1] or elliptical [3] cross section. Further, ξ_0 increases with γ and tends asymptotically to unity for $\gamma \to \infty$.

Interest attaches to the dimensionless flow rate Q^* circulating in a cross section:

$$Q^* = -\int_0^{\xi_0} \frac{u(\xi)}{w_0} d\xi = \int_1^{\xi_0} \frac{u(\xi)}{w_0} d\xi.$$

Figure 3 shows Q^* as a function of γ; the peak value of $Q^* \approx 0.077$ occurs for $\gamma = 3.4$, so the flow rate for the circulation in the cross section does not exceed 8% of the flow in the main direction for any value of γ.

The resistance is characterized by the stress on the wall:

$$\tau = \frac{\nu\rho}{h} \frac{dw}{d\xi}\bigg|_{\xi=1}.$$

We take the ratio of these quantities for the rotating channel (τ_ω) and the immobile one τ_0 for a given w_0 to get

$$\frac{\tau_\omega}{\tau_0} = \frac{8\gamma^3(\operatorname{sh} 2\gamma - \sin 2\gamma)}{3(A^2 + B^2)}, \tag{11}$$

where

$$\tau_0 = -\frac{3\nu\rho w_0}{h}.$$

Figure 3 shows τ_ω/τ_0 as a function of γ; $\tau_\omega \simeq \tau_0$ if γ is less than 1, while for $\gamma > 4$ the result is

$$\tau_\omega/\tau_0 \simeq \gamma^2/3(\gamma - 1). \tag{12}$$

We now consider the heat transfer in a rotating slot channel. We assume that all physical characteristics of the liquid are constant. Steady-state heat transfer applies, and the temperature distribution is independent of the x coordinate. The energy equation can then be put as

$$w\frac{\partial T}{\partial z} = \frac{\lambda}{\rho c_p}\left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \nu\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right]. \tag{13}$$

From (5) and (6) we get that a particular solution to this equation that satisfies the boundary conditions at the wall of the channel

$$T(\pm h) = T_w = T_0 + \sigma z, \quad T_0 = \text{const}, \tag{14}$$

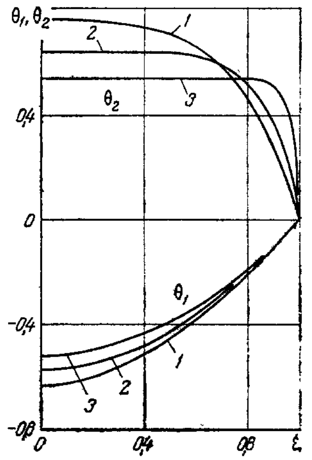

Fig. 4. Graphs for θ_1 and θ_2. 1) $\gamma=0$; 2) 4; 3) 16.

can be put as

$$T - T_w = \frac{\sigma\rho c_p w_0 h^2}{\lambda}\,\theta_1(\xi,\ \gamma) + \frac{\nu\rho w_0^2}{\lambda}\,\theta_2(\xi,\ \gamma), \tag{15}$$

where

$$\theta_1 = \frac{C}{2}\left[\left(\frac{A}{\gamma^2}+B\right)\operatorname{ch}\gamma\cos\gamma + \left(A-\frac{B}{\gamma^2}\right)\operatorname{sh}\gamma\sin\gamma - \xi^2(A\operatorname{sh}\gamma\sin\gamma + B\operatorname{ch}\gamma\cos\gamma) - \frac{A}{\gamma^2}\operatorname{ch}(\gamma\xi)\cos(\gamma\xi) + \frac{B}{\gamma^2}\operatorname{sh}(\gamma\xi)\sin(\gamma\xi)\right]; \tag{16}$$

$$\theta_2 = \frac{4\gamma^2}{A^2+B^2}\,[\operatorname{ch}2\gamma + \cos 2\gamma - \operatorname{ch}(2\gamma\xi) - \cos(2\gamma\xi)]. \tag{17}$$

The solution (15) corresponds to a developed temperature distribution provided that the density of the heat flux is constant on the planes forming the slot; there are two components. The first on the left in (15) is due to convective heat transfer, while the second is due to the heat arising from the dissipation of mechanical energy when the liquid moves along the channel.

Equations (16) and (17) simplify considerably if γ is small:

$$\left.\begin{aligned} \theta_1 &= [\xi^2(6-\xi^2)-5]/8 + O(\gamma^4), \\ \theta_2 &= \frac{3}{4}(1-\xi^4) + O(\gamma^4). \end{aligned}\right\} \tag{18}$$

If γ is very large ($\gamma \to \infty$),

$$\theta_1 \simeq (\xi^2-1)/2, \quad \theta_2 = \{1-\exp[-2\gamma(1-\xi)]\}/2. \tag{19}$$

Comparison of (18) and (19) shows that in this case the rotation has comparatively little effect on the convective heat transfer but causes a considerable flattening of the dissipative temperature distribution in the core. Figure 4, which shows results from (16) and (17) for γ of 0, 4, and 16, illustrates this.

We now calculate the Nusselt number for the case where the convective term in (15) exceeds the dissipative one substantially; the definition

$$\mathrm{Nu} = 2hq_w/\lambda(T_w - T_m)$$

and the heat balance for the direction of the z axis imply that

$$\mathrm{Nu} = 2\Big/\int_0^1 \frac{\theta_1 w}{w_0}\,d\xi. \tag{20}$$

Figure 3 shows results from (20); the behavior of τ_ω/τ_0 and Nu alters considerably when γ is large, since τ_ω/τ_0 increases monotonically with γ, whereas the Nusselt number tends to a limiting value of 6.

NOTATION

A, B, C, constants of integration; h, halfheight of channel; x, y, z, Cartesian coordinate system; Π, modified pressure; u, w, velocity components along the x and z axes; Q, flow rate in the z direction; w_0, mean flow-rate velocity; α, β, Π_0, constants in the pressure function; γ, rotation parameter; ξ, dimensionless coordinate; ν, kinematic viscosity; ρ, density; τ_ω, shear stress on wall of rotating channel; τ_0, stress on fixed wall; T, temperature; q_w, heat flux; λ, thermal conductivity of liquid; c_p, specific heat at constant pressure; T_w, T_m, wall temperature and mean-mass temperature, respectively; Nu, Nusselt number.

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INFLUENCE OF INHOMOGENEOUS ELECTRIC AND MAGNETIC FIELDS ON INTERNAL MASS TRANSFER IN CAPILLARY-POROUS BODIES

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Equations are derived for mass transfer in inhomogeneous electric and magnetic fields. Experimental results are given in support of the theoretical conclusions.

It has been shown experimentally [1, 2] that inhomogeneous electric and magnetic fields have an appreciable influence on internal mass-transfer processes in porous bodies. We wish to examine some possible physical mechanisms of this phenomenon.

It is generally known that dipolar molecules in an inhomogeneous electric field with gradient ∇E are acted upon by a force

$$f = p_e \nabla E, \text{ dyn.} \tag{1}$$

Under the action of this force dipolar molecules acquire a velocity component in the direction of increasing values of ∇E with a magnitude

$$U = Df/kT, \text{ cm/sec.} \tag{2}$$

To the diffusion flux q_v of vapor molecules in this case is added a convective flux $q_e = UC$. The total flux is then

$$q = q_v + q_e = -D\frac{dC}{dx}\left[1 + \frac{p_e C \nabla E}{kT(-dC/dx)}\right]. \tag{3}$$

It is evident from this equation that for $\nabla E > 0$ the vapor transfer rate increases. The influence of the field is particularly appreciable for small vapor-pressure gradients, such that $q_e \gg q_v$, and for molecules with a large dipole moment.

An inhomogeneous field also affects a liquid dielectric, pulling it into the zone of greater field inhomogeneity. The force acting on unit volume of the dielectric is

$$P_e = \frac{\varepsilon - 1}{8\pi}\nabla(E^2), \quad \text{dyn/cm}^3. \tag{4}$$

Under the action of the force per unit volume, P_e, viscous flow is analogous to flow at a constant hydrostatic pressure gradient ∇P. For example, in the case of a cylindrical capillary of radius r the mass flux can be written in the form

$$q = q_l + q_e = \frac{\rho r^2}{8\eta}\left[\nabla P + \frac{(\varepsilon - 1)\nabla(E^2)}{8\pi}\right]. \tag{5}$$

Here the first term expresses the mass flux under the influence of the hydrostatic pressure gradient ∇P, and the second term under the influence of the field gradient ∇E.

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